Period 1, Oct 14, 2024

5. Find $\frac{d^2y}{dx^2}$ in terms of x and y if $xy + y^2 = 4$.

$$xy + y^2 = 4$$

$$1\cdot y + x\cdot\frac{dy}{dx} + 2y\frac{dy}{dx} = 0$$

$$x\frac{dy}{dx} + 2y\frac{dy}{dx} = -y$$

$$\frac{\frac{dy}{dx}\cancel{(x+2y)}}{\cancel{(x+2y)}} = \frac{-y}{(x+2y)}$$

$$\frac{2xy + 2y^2}{2y(x+y)}$$

$$\frac{dy}{dx} = \frac{-y}{(x+2y)}$$

$$\frac{d^2y}{dx^2} = \frac{-\frac{dy}{dx}(x+2y) - (-y)(1 + 2\frac{dy}{dx})}{(x+2y)^2}$$

$$\frac{d^2y}{dx^2} = \frac{-\left(\frac{-y}{\cancel{x+2y}}\right)\cancel{(x+2y)} + y\left(1 + 2\cdot\frac{-y}{x+2y}\right)}{(x+2y)^2}$$

$$\frac{d^2y}{dx^2} = \frac{y + y + \frac{-2y^2}{x+2y}}{(x+2y)^2} = \frac{\frac{2y(x+2y)}{1(x+2y)} - \frac{2y^2}{x+2y}}{(x+2y)^2}$$

$$\frac{d^2y}{dx^2} = \frac{\frac{2xy + 4y^2 - 2y^2}{(x+2y)}}{(x+2y)^2} = \frac{\frac{2y(x+y)}{(x+2y)}}{(x+2y)^2}$$

$$\frac{d^2y}{dx^2} = \frac{2y(x+y)}{(x+2y)}\cdot\frac{1}{(x+2y)^2}$$

$$\frac{d^2y}{dx^2} = \frac{2y(x+y)}{(x+2y)^3}$$

3. Find the slope of the curve at the given point.

a. $\sqrt{xy} = 1$ at $\left(2, \frac{1}{2}\right)$.

$$(xy)^{\frac{1}{2}} = 1$$

$$\frac{1}{2}(xy)^{\frac{1}{2}-1}\left[1\cdot y + x\cdot\frac{dy}{dx}\right] = 0$$

$$\frac{1}{2\sqrt{xy}}\left[y + x\frac{dy}{dx}\right] = 0$$

$$\frac{y + x\frac{dy}{dx}}{2\sqrt{xy}} = 0$$

$$\frac{\frac{1}{2} + 2\cdot\frac{dy}{dx}}{2\sqrt{2\cdot\frac{1}{2}}} = \frac{\frac{1}{2} + 2\frac{dy}{dx}}{2} = 0$$

$$\frac{\frac{1}{2}}{\frac{2}{1}} + 2\frac{dy}{dx} = 0 \Rightarrow$$

$$\Rightarrow \frac{y}{2\sqrt{xy}} + \frac{x}{2\sqrt{xy}}\frac{dy}{dx} = 0 \quad -\frac{y}{2\sqrt{xy}}$$

$$\frac{2\sqrt{xy}}{x}\cdot\frac{x}{2\sqrt{xy}}\frac{dy}{dx} = \frac{-y}{\cancel{2\sqrt{xy}}}\cdot\frac{\cancel{2\sqrt{xy}}}{x}$$

$$\frac{dy}{dx} = -\frac{y}{x} = \frac{-\frac{1}{2}}{\frac{2}{1}} = -\frac{1}{4}$$

$$m = -\frac{1}{4}$$

d) $\sin^2 y + \cos^2 y = y + 2$

$1 = y + 2$

$0 = \frac{dy}{dx} + 0$

$\frac{dy}{dx} = 0$

b. $y + \sin y + x^2 = 9$ at $(3, 0)$.

$\frac{dy}{dx} + \cos y \frac{dy}{dx} + 2x = 0$

$\frac{dy}{dx} + (\cos 0)\frac{dy}{dx} + 2(3) = 0$

$\frac{dy}{dx} + 1\frac{dy}{dx} = -6$

$2\frac{dy}{dx} = -6$

$\frac{dy}{dx} = -3$

Point $(3,0)$ $m = -3$

$y - y_1 = m(x - x_1)$

$y - 0 = -3(x-3)$

$y = -3x + 9$

Tangent Line

Normal line

Point $(3,0)$ $m = -\left(\frac{1}{-3}\right) = \frac{1}{3}$

$y - 0 = \frac{1}{3}(x-3)$

$y = \frac{1}{3}x - 1$

b. $x^3 + \underbrace{5x^2}\underbrace{y} + 2y^2 = 4y + 11$ at $(1, 2)$.

$3x^2 + 10x \cdot y + 5x^2 \cdot \frac{dy}{dx} + 2 \cdot 2y^{2-1}\frac{dx}{dx} = 4\frac{dy}{dx} + 0$

$3x^2 + 10xy + \cancel{5x^2\frac{dy}{dx}} + \cancel{4y\frac{dy}{dx}} = 4\frac{dy}{dx}$

$\cancel{-5x^2\frac{dy}{dx}} \quad \cancel{-4y\frac{dy}{dx}} \quad -5x^2\frac{dy}{dx} - 4y\frac{dy}{dx}$

$3x^2 + 10xy = \frac{dy}{dx}(4 - 5x^2 - 4y)$

$3(1^2) + 10(1 \cdot 2) = \frac{dy}{dx}[4 - 5(1)^2 - 4(2)]$

$23 = \frac{dy}{dx}[4 - 5 - 8] = \frac{dy}{dx}[-9]$

$-2\frac{5}{9} = -\frac{23}{9} = \frac{dy}{dx}$

7. Find the points where the graph of $x^2 + 3y^2 = 7$ has a vertical tangent line.

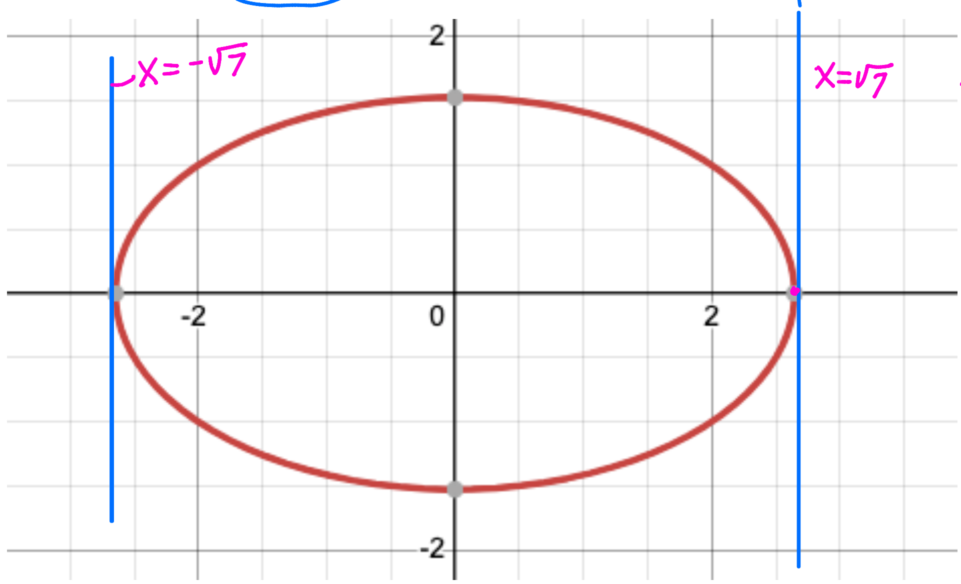
$2x + 6y\frac{dy}{dx} = 0 \Rightarrow 6y\frac{dy}{dx} = -2x \Rightarrow \frac{dy}{dx} = \frac{-x}{3y} \Rightarrow m = \frac{a}{0}$

$y = 0$

$x^2 + 3(0)^2 = 7$

$x^2 = 7$

$x = \pm\sqrt{7}$



1) $F(x) = 7x^4 - 3x^5 + 6x^3 - 3e^x$

$F'(x) = 28x^3 - 15x^4 + 18x^2 - 3e^x$

$F(x) = 5x^4 - 3x^8 + 6x^2 - e^x$

$F'(x) = 20x^3 - 24x^7 + 12x - e^x$

2) $F(x) = \dfrac{5x^3 + 7x^{\frac{1}{2}} - 4x}{\sqrt[4]{x}}$

$F'(x) = \dfrac{(15x^2 + \frac{7}{2}x^{-\frac{1}{2}} - 4)\sqrt[4]{x} - (5x^3 + 7x^{\frac{1}{2}} - 4x)(\frac{1}{4}x^{-\frac{3}{4}})}{(\sqrt[4]{x})^2}$

$\dfrac{(15x^2 + \frac{7}{2\sqrt{x}} - 4)\sqrt[4]{x} - (\frac{1}{4\sqrt[4]{x^3}})(5x^3 + 7\sqrt{x} - 4x)}{\sqrt{x}}$

$$\frac{15x^{2\frac{1}{4}} + \frac{7}{2}x^{-\frac{1}{4}} - 4\cancel{x}^{\frac{1}{4}}}{x^{\frac{1}{2}}} + \frac{\frac{5x^3+7\sqrt{x}-4x}{4\sqrt[4]{x^3}}}{\frac{\sqrt{x}}{1}}$$

$$\frac{15x^{2\frac{1}{4}}}{\sqrt{x}} + \frac{7}{2\sqrt[4]{x^3}} - \frac{4}{\sqrt[4]{x}} + \frac{5x^3+7\sqrt{x}-4x}{\sqrt{x}\left(4\sqrt[4]{x^3}\right)}$$

$$F(x) = \frac{4x^3+6x^{\frac{1}{2}}-x}{\sqrt[3]{x}} = \frac{4x^3-6x^{\frac{1}{3}}-x}{x^{\frac{1}{3}}} = \frac{4x^3}{x^{\frac{1}{3}}} - 6\frac{x^{\frac{1}{3}}}{x^{\frac{1}{3}}} - \frac{x}{x^{\frac{1}{3}}}$$

$$= 4x^{2\frac{2}{3}} - 6x^{\frac{1}{6}} - x^{\frac{2}{3}}$$

$$F'(x) = \frac{\left[12x^2 - 6\cdot\frac{1}{2}x^{-\frac{1}{2}} - 1\right]\left[x^{\frac{1}{3}}\right] - \left[4x^3 - 6x^{\frac{1}{3}} - x\right]\left[\frac{1}{3}x^{-\frac{2}{3}}\right]}{\sqrt[3]{x^2}}$$

$$\frac{12x^{2\frac{1}{2}} + 3 - \sqrt{x}}{\left(\sqrt[3]{x}\right)^2 \cdot \sqrt{x}} - \frac{4x^3+6\sqrt{x}-x}{3x^{4/3}}$$

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3) $F(x) = (8x^{-3}+2x^5)(3x^6-7x^2+5)$

$F'(x) = (-24x^{-4}+10x^4)(3x^6-7x^2+5) + (8x^{-3}+2x^5)(18x^5-14x+0)$

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3) $F(x) = (12x^{-2}+6x^3)(8x^5-7x^2+3)$

$F'(x) = (-24x^{-3}+18x^2)(8x^5-7x^2+3) + (12x^{-2}+6x^3)(40x^4-14x+0)$

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4)

$$F(x) = \frac{[(4x^2+2)(7x^4-3x^{-2})]}{(2x^3+1)}$$

$$\underbrace{28x^6 - 12 + 14x^4 - 6x^{-2}} \Rightarrow \underbrace{168x^5 + 56x^3 + 12x^{-3}}$$

$$F'(x) = \frac{\overbrace{[8x(7x^4-3x^{-2}) + (4x^2+2)(28x^3+6x^{-3})]}(2x^3+1) - \overbrace{(4x^2+2)(7x^4-3x^{-2})}(6x^2)}{(2x^3+1)^2}$$

4.

$$F(x) = \frac{(9x^2+4)(7x^5-3x^{-1})}{(2x^3+7)} = \frac{63x^7 - 27x + 28x^5 - 12x^{-1}}{(2x^3+7)}$$

$$F'(x) = \frac{(441x^6 - 27 + 140x^4 + 12x^{-2})(2x^3+7) - (63x^7 - 27x + 28x^5 - 12x^{-1})(6x^2)}{(2x^3+7)^2}$$

$$F'(x) = \frac{[18x(7x^5-3x^{-1}) + 9(x^2+4)(35x^4+3x^{-2})](2x^3+7) - (9x^4+4)(7x^5-3x^{-1})(6x^2)}{(2x^3+7)^2}$$